

APPARENT VERSUS TRUE VALUE OF THE COSMOLOGICAL CONSTANT

Antonio Enea Romano^{1,2,3*} and Pisin Chen^{2,3,4,5†}

¹*Instituto de Fisica,*

Universidad de Antioquia,

A.A.1226, Medellin, Colombia.

²*Department of Physics,*

National Taiwan University,

Taipei 10617, Taiwan, R.O.C.

³*Leung Center for Cosmology and Particle Astrophysics,*

National Taiwan University,

Taipei 10617, Taiwan, R.O.C.

⁴*Graduate Institute of Astrophysics,*

National Taiwan University,

Taipei 10617, Taiwan, R.O.C.

⁵*Kavli Institute for Particle Astrophysics and Cosmology,*

SLAC National Accelerator Laboratory,

Menlo Park, CA 94025, U.S.A.

Abstract

Supernovae observations strongly support the presence of a cosmological constant, but its value, which we will call apparent, is normally determined assuming that the Universe can be accurately described by a homogeneous model. Even in the presence of a cosmological constant we cannot exclude nevertheless the presence of a small local inhomogeneity which could affect the apparent value of the cosmological constant. Neglecting the presence of the inhomogeneity can in fact introduce a systematic misinterpretation of cosmological data, leading to the distinction between an apparent and the true value of the cosmological constant. But is such a difference distinguishable? Recently we set out to model the local inhomogeneity with a Λ *LTB* solution and computed the relation between the apparent and the true value of the cosmological constant. In this essay we reproduce the essence of our model with the emphasis on its physical implications.

*Electronic address: aer@phys.ntu.edu.tw

†Electronic address: pisinchen@phys.ntu.edu.tw

I. INTRODUCTION

High redshift luminosity distance measurements [1–6] and the WMAP measurements [7, 8] of cosmic microwave background (CMB) interpreted in the context of standard FLRW cosmological models strongly disfavor a matter dominated universe and strongly support a dominant dark energy component, which gives rise to an accelerated expansion of the universe.

One of the main assumptions of standard cosmology is that the metric describing space time on a sufficiently large scale is homogeneous, but this is more a simplifying theoretical hypothesis than an actual observational conclusive result. All the cosmological parameters whose apparent value is estimated under this homogeneity assumption may have different true values, if the Universe is actually inhomogeneous. The value of the cosmological constant for example could be different from the one which is obtained from fitting data with a homogeneous FLRW metric as it is common practice with the Λ CDM models, even in presence of relatively small large scale inhomogeneities. This type of effect would be more important for local inhomogeneities which surround the observer, and the first step towards taking them into account is to consider the effect of spherically symmetric inhomogeneities. A more general treatment would involve to include the effects of less symmetric cases, such as not central observers or not spherically symmetric spaces.

This type of space time geometry has already received a lot of attention in a cosmological context. As an alternative to dark energy, it has in fact been proposed [9, 10] that we may be at the center of an inhomogeneous isotropic universe without cosmological constant, as described by a Lemaitre-Tolman-Bondi (LTB) solution of Einstein’s field equations. Interesting analysis of observational data in inhomogeneous models without dark energy and of other theoretically related problems are given, for example, in [11–35]

Recently we have adopted a different approach [36]. We considered a Universe that consists of a cosmological constant and matter with some local large scale inhomogeneity. We modeled this by a Λ LTB solution. In this essay we will reproduce the essence of this model with the emphasis on its implications. For simplicity we will assume that we are located at the center of this local inhomogeneity. In this regard, this can be considered a first attempt to model local large scale inhomogeneities in the presence of the cosmological constant or, more generally, dark energy.

After calculating the null radial geodesics for a central observer we then compute the luminosity distance and compare it to that of Λ CDM model, finding the relation between the two

different cosmological constants appearing in the two models, where we call apparent the one in the Λ CDM and true the one in Λ LTB. Our calculations show that the corrections to Ω_{Λ}^{app} , which is the value of the cosmological constant obtained from analyzing supernovae data by assuming homogeneity, can be important and should be taken into account.

II. LTb SOLUTION WITH A COSMOLOGICAL CONSTANT

The LTb solution can be written as [37–39]

$$ds^2 = -dt^2 + \frac{(R_{,r})^2 dr^2}{1 + 2E(r)} + R^2 d\Omega^2, \quad (1)$$

where R is a function of the time coordinate t and the radial coordinate r , $E(r)$ is an arbitrary function of r , and $R_{,r} = \partial_r R(t, r)$.

The Einstein equations with dust and a cosmological constant give

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{2E(r)}{R^2} + \frac{2M(r)}{R^3} + \frac{\Lambda}{3}, \quad (2)$$

$$\rho(t, r) = \frac{2M_{,r}}{R^2 R_{,r}}, \quad (3)$$

with $M(r)$ being an arbitrary function of r , $\dot{R} = \partial_t R(t, r)$ and $c = 8\pi G = 1$ is assumed throughout this essay.

The general analytical solution for a FLRW model with dust and cosmological constant was obtained by Edwards [40] in terms of the elliptic functions. Inspired by the FLRW case, we can construct a general solution of the partial differential equation Eq.(2). First, we introduce a new coordinate $\eta = \eta(t, r)$ and a variable a by

$$\left(\frac{\partial \eta}{\partial t}\right)_r = \frac{r}{R} \equiv \frac{1}{a}, \quad (4)$$

and introduce new functions by

$$\rho_0(r) \equiv \frac{6M(r)}{r^3}, \quad k(r) \equiv -\frac{2E(r)}{r^2}. \quad (5)$$

Then Eq.(2) becomes

$$\left(\frac{\partial a}{\partial \eta}\right)^2 = -k(r)a^2 + \frac{\rho_0(r)}{3}a + \frac{\Lambda}{3}a^4, \quad (6)$$

where a is now regarded as a function of η and r ; $a = a(\eta, r)$. It should be noted that the coordinate η , which is a generalization of the conformal time in a homogeneous FLRW universe,

has been only implicitly defined by Eq.(4). The actual relation between t and η can be obtained by integrating $t = \int a d\eta$ once $a(\eta, r)$ is known.

Inspired by the construction of the solution for the FLRW case, we can now set

$$a(\eta, r) = \frac{\alpha}{3\phi(\frac{\eta}{2L}; g_2, g_3) + kL^2}, \quad (7)$$

which leads to the Weierstrass differential equation for the choice of the parameters given by

$$\alpha = \rho_0(r)L^2, \quad g_2 = \frac{4}{3}k(r)^2L^4, \quad g_3 = \frac{4}{27}(2k(r)^3 - \Lambda\rho_0(r)^2)L^6, \quad (8)$$

where we have introduced the length L for dimensional consistency. $\phi(x; g_2, g_3)$ is the Weierstrass elliptic function satisfying the differential equation,

$$\left(\frac{d\phi}{dx}\right)^2 = 4\phi^3 - g_2\phi - g_3. \quad (9)$$

We finally get

$$a(\eta, r) = \frac{\rho_0(r)L^2}{3\phi\left(\frac{\eta}{2L}; g_2(r), g_3(r)\right) + k(r)L^2}. \quad (10)$$

In this essay we will set $L = (a_0 H_0)^{-1}$ and choose the so called FLRW gauge, i.e. the coordinate system in which $\rho_0(r)$ is constant.

III. CALCULATING THE LUMINOSITY DISTANCE

We adopt the same method developed in [41] to solve the null geodesic equation written in terms of the coordinates (η, r) . The luminosity distance for a central observer in the LTB space-time as a function of the redshift z is expressed as

$$D_L(z) = (1+z)^2 R(t(z), r(z)) = (1+z)^2 r(z) a(\eta(z), r(z)), \quad (11)$$

where $(t(z), r(z))$ or $(\eta(z), r(z))$ is the solution of the radial geodesic equation as a function of z . Using the analytical solution we can derive the geodesics equations:

$$\frac{d\eta}{dz} = -\frac{\partial_r t(\eta, r) + F(\eta, r)}{(1+z)\partial_\eta F(\eta, r)} \equiv p(\eta, r), \quad (12)$$

$$\frac{dr}{dz} = \frac{a(\eta, r)}{(1+z)\partial_\eta F(\eta, r)} \equiv q(\eta, r), \quad (13)$$

where

$$F(\eta, r) \equiv \frac{R_{,r}}{\sqrt{1+2E(r)}} = \frac{1}{\sqrt{1-k(r)r^2}} \left[\partial_r(a(\eta, r)r) - a^{-1}\partial_\eta(a(\eta, r)r) \partial_r t(\eta, r) \right]. \quad (14)$$

It is important that the functions p, q, F have explicit analytical forms.

In order to obtain the luminosity distance as a function of the redshift, we to use the following expansions:

$$k(r) = k_0 + k_1 r + k_2 r^2 + \dots \quad (15)$$

$$t(\eta, r) = b_0(\eta) + b_1(\eta)r + b_2(\eta)r^2 + \dots \quad (16)$$

Since we are interested in the effects due to the inhomogeneities, we will neglect k_0 in the rest of the calculation because this corresponds to the homogeneous component of the curvature function $k(r)$. Following the same approach given in [31], we take local Taylor expansion in redshift for the geodesic equations, and find the luminosity distance as follows:

$$D_L^{\Lambda LTB}(z) = (1+z)^2 r(z) a^{\Lambda LTB}(\eta(z), r(z)) = D_1^{\Lambda LTB} z + D_2^{\Lambda LTB} z^2 + D_3^{\Lambda LTB} z^3 + \dots \quad (17)$$

$$D_1^{\Lambda LTB} = \frac{1}{H_0},$$

$$D_2^{\Lambda LTB} = \frac{1}{36 H_0 (\Omega_\Lambda^{true} - 1)} (54 B_1 (\Omega_\Lambda^{true} - 1)^2 + 18 B_1' (\Omega_\Lambda^{true} - 1) - 18 h_{0,r} (\Omega_\Lambda^{true})^2 + 30 h_{0,r} \Omega_\Lambda^{true} + -12 h_{0,r} + 6 K_1 \Omega_\Lambda^{true} - 10 K_1 + 27 (\Omega_\Lambda^{true})^2 - 18 \Omega_\Lambda^{true} - 9), \quad (18)$$

where we have introduced the dimensionless quantities $K_0, K_1, B_1, B_1', h_{0,r}$ according to

$$H_0 = \left(\frac{\partial_t a(t, r)}{a(t, r)} \right)^2 \Big|_{t=t_0, r=0} = \left(\frac{\partial_\eta a(\eta, r)}{a(\eta, r)^2} \right)^2 \Big|_{\eta=\eta_0, r=0}, \quad (19)$$

$$K_0 = k_0 (a_0 H_0)^{-2}, \quad (20)$$

$$K_1 = k_1 (a_0 H_0)^{-3}, \quad (21)$$

$$B_1(\eta) = b_1(\eta) a_0^{-1}, \quad (22)$$

$$B_1 = b_1(\eta_0) a_0^{-1}, \quad (23)$$

$$B_1' = \frac{\partial B_1(\eta)}{\partial \eta} \Big|_{\eta=\eta_0} (a_0 H_0)^{-2}, \quad (24)$$

$$h_{0,r} = \frac{1}{a_0 H_0} \frac{\partial_r a(\eta, r)}{a(\eta, r)} \Big|_{\eta=\eta_0, r=0}, \quad (25)$$

$$t_0 = t(\eta_0, 0), \quad (26)$$

and used the Einstein equation at the center ($\eta = \eta_0, r = 0$) with

$$1 = \Omega_k^0(0) + \Omega_M^0 + \Omega_\Lambda, \quad (27)$$

$$\Omega_k^0(r) = -\frac{k(r)}{H_0^2 a_0^2}, \quad (28)$$

$$\Omega_M^0 = \frac{\rho_0}{3H_0^2 a_0^3}, \quad (29)$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}. \quad (30)$$

In order to put the formula for the luminosity distance in this form it is necessary to manipulate appropriately the elliptic functions and then reexpress everything in terms of physically meaningful quantities such as H_0 .

For a FLRW space time we can calculate the luminosity distance using the following relation, which is only valid assuming flatness.

$$D_L^{\Lambda CDM}(z) = (1+z) \int_0^z \frac{dz'}{H^{\Lambda CDM}(z')} = D_1^{\Lambda CDM} z + D_2^{\Lambda CDM} z^2 + D_3^{\Lambda CDM} z^3 + \dots \quad (31)$$

From which we can get

$$D_1^{\Lambda CDM} = \frac{1}{H_0}, \quad (32)$$

$$D_2^{\Lambda CDM} = \frac{3\Omega_\Lambda^{app} + 1}{4H_0}. \quad (33)$$

IV. APPARENT AND TRUE VALUES OF THE COSMOLOGICAL CONSTANT

So far we have calculated the first two terms of the redshift expansion of the luminosity distance for ΛLTB and ΛCDM models. Since we know that the latter provides a good fitting for supernovae observations, we can now look for the ΛLTB models which give the same theoretical prediction. In order to find the relation between the apparent and the true value of the cosmological constant, we need in fact to match the terms in the redshift expansion, i.e.,

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB} \quad , \quad 1 \leq i \leq 2. \quad (34)$$

From the above relations we find

$$H_0^{\Lambda LTB} = H_0^{\Lambda CDM}, \quad (35)$$

$$\Omega_\Lambda^{app} = \frac{1}{27(\Omega_\Lambda^{true} - 1)} \left[54B_1(\Omega_\Lambda^{true})^2 - 108B_1\Omega_\Lambda^{true} + 54B_1 + 18B_1'\Omega_\Lambda^{true} - 18B_1' - 18h_{0,r}(\Omega_\Lambda^{true})^2 + 30h_{0,r}\Omega_\Lambda^{true} - 12h_{0,r} + 6K_1\Omega_\Lambda^{true} - 10K_1 + 27\Omega_\Lambda^{true}(\Omega_\Lambda^{true} - 1) \right], \quad (36)$$

$$\Omega_\Lambda^{true} = -\frac{1}{6(6B_1 - 2h_{0,r} + 3)} \left[\left((36B_1 - 6B_1' - 10h_{0,r} - 2K_1 + 9\Omega_\Lambda^{true} + 9)^2 + \right. \right.$$

$$\begin{aligned}
& -4(6B_1 - 2h_{0,r} + 3)(54B_1 - 18B'_1 - 12h_{0,r} - 10K_1 + 27\Omega_\Lambda^{true}) \Big)^{1/2} - 36B_1 \\
& + 6B'_1 + 10h_{0,r} + 2K_1 - 9(\Omega_\Lambda^{true} - 1) \Big]. \tag{37}
\end{aligned}$$

We can also expand the above exact relations by assuming that all the inhomogeneities can be treated perturbatively with respect to Λ CDM, i.e., $\{K_1, B_1, B'_1\} \propto \epsilon$, where ϵ stands for a small deviation from the *FLRW* solution:

$$\begin{aligned}
\Omega_\Lambda^{true} = \Omega_\Lambda^{app} & - \frac{2}{27(\Omega_\Lambda^{app} - 1)} (27B_1(\Omega_\Lambda^{app} - 1)^2 + 9B'_1(\Omega_\Lambda^{app} - 1) - 9h_{0,r}(\Omega_\Lambda^{app})^2 + 15h_{0,r}\Omega_\Lambda^{app} \\
& - 6h_{0,r} + 3K_1\Omega_\Lambda^{app} - 5K_1) + O(\epsilon^2). \tag{38}
\end{aligned}$$

As expected, all these relations reduce to

$$\Omega_\Lambda^{true} = \Omega_\Lambda^{app}, \tag{39}$$

in the limit in which there is no inhomogeneity, i.e. when $K_1 = B_1 = B'_1 = h_{0,r} = 0$.

V. CONCLUSIONS

We have derived for the first time the correction due to local large scale inhomogeneities to the value of the apparent cosmological constant inferred from low redshift supernovae observations. This analytical calculation shows how the presence of a local inhomogeneity can affect the estimation of the value of cosmological parameters, such as Ω_Λ . This effect should be properly taken into account both theoretically and observationally.

While this should be considered only as the first step towards a full inclusion of the effects of large scale inhomogeneities in the interpretation of cosmological observations, it is important to emphasize that we have introduced a general definition of the concept of apparent and true value of cosmological parameters, and shown the general theoretical approach to calculate the corrections to the apparent values obtained under the standard assumption of homogeneity.

Acknowledgments

This research is supported by Taiwan National Science Council under project No. NSC 97-2112-M-002-026-MY3 and by US department of Energy under Contract No. DE-AC03-

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